# Attempt at Unification of Interactions and Quantisation of Gravitation

Zygmunt Morawski

ABSTRACT: Strating with the improved conception of Dirac's unempty vacuum [1] the easy dependence creating the fundaments of unification of interaction was introduced. Using the Einstein's equation of gravitation and Dirac's equation the procedure of quantisation of gravitation was committed.
It led to explanation of the quasiquantum Titius-Body's law and to taking under consideration the conception of supersymmetric particles.
The 8-dimensional spacetime was considered consequently with generalization of tensor occuring in Einstein's equation and matrices in Dirac's equation.
The Dirac's eqaution was generalised to describe aach value of projection of spin $s=\frac{m}{2}$ ; $m \in N \cup \{0\}$.

## I

## Introduction

1.1. During the last twenty years a lot of interesting ideas concerning unification of interactions and quantisations of gravitation occured.

E.Witten introduced the conception of strings, modes of oscilation of which correspond with particles, and energy of these modes correpends with particles [2] .

E.Witten refered to the Kaluza-Klein conception [3,4]. In his works he considered the 10-dimensional and 11-dimensional spaces.

Next M.J.Duff considered the membranes in 11-dimensional spacetime analysing the model of oscilations of membranes [5-7].

These works were continuation of works from the turn of 70th and 80th years, when numerous authors, for example J.Goldstone analysed even 26-dimensional spaces. [8-10]

With the tendency to increasing of number of dimensions occured the tendency to reduction - B.Julia, E.Cremmer [11,12] E.Witten [4].

This work concerns the 8-dimensional space, which may be the subspace of, for example, 11-dimensional space. The author is convinced, that the reality surrounding us has more than 4 dimensions, but values the results of works, authors of which tend to decreasing of number of dimensions.

There is no discrepancy; the space characterized by smaller number of dimensions may be included in the space characterized by bigger number of dimensions and the results may be generalized.

The works concerning the 4-dimensional spacetime are worth attention because of this reason.

The conception of spacetime loops presented in the work by A.Ashtekar and others [13] is specially interesting in context of quantisation of gravitation. This same author has advertised in another work [14] the appearing of negative values of self energy, which is used in this work.

Next Y.Bars in his works [15] developed differential geometry in loops space and made correlation of geometry and gravitation.

In this work the conncetion of geometry and mass and charge was made by enlarging the energy momentum tensor existing in Einstein's equation, by adding the tensor component, called charge mass tensor (Chapter III).

The educed easy dependence of mass on charge became the fundament of considerations the unification of interactions (Chapter II).

Moreover, in the framework of procedure of quantisation of gravitation, the possibility of investigation of wave function (in spinar form) in curved multidimensional spacetime arose (Chapeer III).

This work is a continuation of work [1]. The conception of complex mass, detailly discussed in those work (which more or less directly had occured in earlier works [16-19]) was used.

The possibilities of further developement of presented concpetions are described in chapter IV.

## II

## Unification of interactions

2.1. The start point of consideration is the problem of negative mass existing in different physical problems. And so we have negative values of self energy in quantum gravity [14] which, on the strenght of equation $E=mc^2$ corresponds with negative mass.

The next known case is negative effective mass of electron in solid body. [20]

The important fact in further considerations will be the negative mass of particles in Dirac's sea. [21]

Let's analyse the equation:

$$E^2 = p^2c^2 + m_o^2c^4 \tag{1}$$

from which we obtain two matrices of energy: positive and negative

$$E_{1/2} = \pm\sqrt{p^2c^2 + m_o^2c^4}$$

and in this way we obtain two possibilities:

$$E_1 = m_1c^2 \ ; \quad m_1 > 0 \quad \text{and} \quad E_2 = m_2c^2 \ ; \quad m_2 < 0 \tag{2}$$

Analogically we have two cases:

$$E_3 = m_oc^2 \ ; \quad m_o > 0 \quad \text{and} \quad E_4 = m_oc^2 \ ; \quad m_o < 0 \tag{3}$$

"Bare" mass $m_o$ may have both positive and negative sign, the second power of mass is in both cases the same.

Let's take the formula under consideration [21] :

$$\frac{\partial}{\partial t}\left[\frac{i\hbar}{2mc^2}\left(\psi^x\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^x}{\partial t}\right)\right] + \operatorname{div}\frac{\hbar}{2mi}\left[\psi^x(\nabla\psi) - \psi(\nabla\psi^x)\right] = 0$$

The quantity:

$$\rho = \frac{i\hbar}{2mc^2}\left(\psi^x \frac{\partial\psi}{\partial t} - \psi \frac{\partial\psi^x}{\partial t}\right)$$

ought to have the sense of density of probability.

Let's take $\psi = Ae^{i\omega t}$

Then

$$\rho = AA^x(2i\omega)\left(\frac{ik}{2mc^2}\right) = AA^x \frac{k\omega}{mc^2} \qquad (5)$$

$$AA^x = |A|^2 \geqslant 0$$

and $\rho$ is positively determined then and only then, when $m < 0$ (compare the second and fourth case in formulas (2) and (3)).

This fact means, that it is possible and even necessary to unit the conferment of physical sense on negative mass, and the positive determination of one particle density of probability.

2.2. Let's take under consideration the states characterized by negative energy implicated by some solutions of Dirac's equation.

The discrepancies which led to ignoring the conception of unempty vacuum Dirac's, have been removed [1].

Two coupled Dirac's seas for particle and antiparticle have been introduced, but the conception particle-hole has been converted [1].

The state of vacuum in Dirac's attempt is the state with all filled levels characterized by negative energy and all empty levels characterized by positive energy.

The excitation causes that particle and hole arise.

The hole may be interpreted either as the sea of particles with negative charge -e and energy (mass) positive with one unpopulated state, or as one particle with positive charge and negative energy (mass).

So the quotient charge to mass and the sign of this quotient characterising physical particle, is important.

The product of mass and charge may be taken under consideration, but the quotient exists in the formula:

$$\vec{E} = \frac{e}{m} \vec{a} \qquad (6)$$

and has physical meaning.
(E - intensity of electric field, a - acceleration of the particle)

Let's take the formula under consideration:

$$\frac{e}{m} = \beta$$

and transforme it to the shape

$$\frac{m}{e} = \alpha$$

and next:

$$m = \alpha e \qquad (7)$$

On the left side od the formula (7) occurs mass and on the right side electric charge. The coefficient $\alpha$ has the physical meaning.

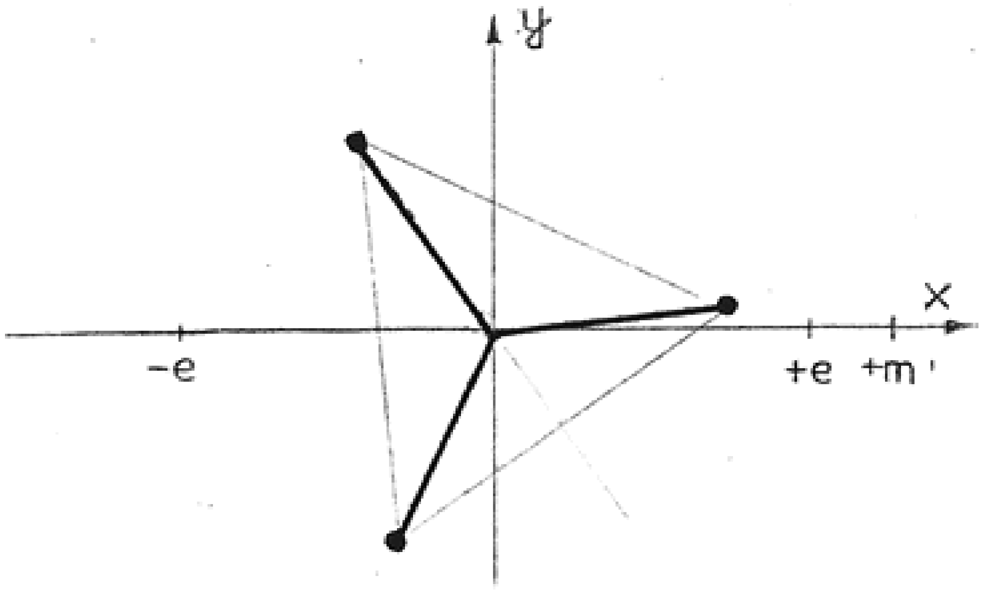
In this way the unification of gravitational and electromagnetic interactions has been achieved.

Mass is positive and equal both in the case of electron and positron. So the equation (7) may be written in the form:

$$m = \alpha |e| \; ; \; \alpha > 0 \qquad (8)$$

The negative mass may be obtained by multiplication of both sides of equation (8) with the factor -1, and relativistic dependence - by multiplication of both sides of equation (8) with the factor $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, and $\alpha$ absorbs this factor.

The module in equation (8) creates the possibility of including of strong interactions into procedure of unification by representation of charges: blue, green, red as the vertices of the equilateral triangle on the complex (number) plane with the centre of symmetry on the origin of coordinates.




Figure

In this way vector binding the origin of coordinates with point representing the strong charge has in each case the same lenght, and complex number describing strong charge has in each case this same absolute value.

In so way, with use of formula (8), the unification of interactions: gravitational, electromagnetic and strong, has been obtained.

The equation (8) has the identical, mathematical shape, as the formula

$E = mc^2$

unifying mass and energy

and identical mathematical shape as equation [22]

$$g = \sin\theta\, e \qquad (9)$$

unifying the weak and electromagnetic interaction.
(g - coupling constant of weak interactions, e - electric charge, $\theta$ - Cabbibo angle).

Naturally the weak interactions are included by equation (9) into equation (8) and in so way go into extend of made great unification.

It is logical that, when interactions are physically equivalent, so then the fields of interactions are physically equivalent, so their sources are physically equivalent, too. It is the physical essence of the equation (8).

The great simplicity of equation (8) unifying all interactions is its salience.

The existence of topological "magnetic" charge [23,24] attest to similar influence of charge and mass on spacetime and attests their physical identity too.

The next fact attests more the identical influence of mass and charge on spacetime.

The body with big mass curves meaningly the trajectory of photon, but it doesn't capture it.
The body with enough great mass curves the trajectory of photon so strongly, that it comes near to this body, on the spiral trajectory.

There is an inte mediate variant too. The body with enough big, but not too big mass, curves the trajectory of photon so that it neither comes nearer nor escapes, but moves on the closed trajectory around this central body.

The photon is caught, but it does not fall on the body.

The situation of captured by massive centre electromagnetic wave (photon) is analogical to the situation of electron wave skirting the positive centre. (Naturally, in both cases the waves aren't attenuated, because the lenght of their orbits is natural multiple of their wavelenghts. In the case of electron the fact comes more, the wavelenght is comparable with the lenght of orbit.)

2.3. Let's notice, that when we ascribe in turn the numbers $n = 1,2,3$ to interactions: gravitational, electromagnetic and strong, then the number of charges of each of these interactions is equal $n$ and is equal the number of $n$ (degree) roots of unity.

So we have:

$$m = \alpha \left| Q \sqrt[n]{1} \right| \quad ; \quad Q \in R_+ \qquad (10)$$

The formula (10) is a generalization of the fact, that each charge has mass. It happens moreover that none charge is bound with mass, as for example in the case of photon. Such mass will be treated as gravitational charge.

Let's discuss the formula (10) for following interactions. Let's introduce the notions of effective mass and absolute mass, connected and definied by the formula:

$$m_{absolute} = \left| m_{effecive} \right|$$

Let's analyse the case of gravitation. In this case, the number of charges (poles) $n=1$ and $m_{absolute} = m_{effective}$ when $n \in R_+ \cup \{0\}$.

Generally $m_{absolute} = |m|$ and in this case are included both cases when $m < 0$ and $m = m_1 + im_2$

($m$ is complex number)

Electromagnetic interaction. Number of poles - n=2.
So we have:

$$m_b = \alpha \left| e \sqrt[2]{1} \right| \qquad (11)$$

so $m_b = \alpha |e|$ or $m_b = \alpha |-e|$

Generally:

$$m_b = \alpha \left| q\, e^{i\left(\varphi + \frac{l}{2} 2\pi\right)} \right| \quad ; \quad l=0,1 \ ; \ \varphi \in R \ , \quad q \in R_+ \qquad (12)$$

($\varphi = 0$ - the latest case)

The formulas (11) and (12) apply to electric charge as well as to hypothetical magnetic one.

Strong interaction. Number of poles n = 3 .

There are three charges: $c \cdot e^{i\varphi}$ , $c \cdot e^{i\left(\varphi + \frac{1}{3} \cdot 2\pi\right)}$ , $c \cdot e^{i\left(\varphi + \frac{2}{3} \cdot 2\pi\right)}$

Generally:

$$m_b = \alpha \left| c \cdot e^{i\left(\varphi + \frac{h}{3} \cdot 2\pi\right)} \right| \qquad c \in R_+ \ ; \ \varphi \in R \ , \ k=0,1,2 \qquad (13)$$

In this case the complex mass is always, even when $\varphi = 0$ .
The formula (13) may be generalized for any number $n$ ($n \in N$) of poles (of charges).

We have:

$$m_{bkn} = \alpha \left| Q \cdot e^{i\left(\varphi + \frac{k}{n} \cdot 2\pi\right)} \right| \qquad (14)$$

n - number of poles
k - number of pole ; k = 0,1,2...k-1
$\varphi \in R$
$Q \in R_+ \cup \{0\}$

$m_{bkn}$ - the absolute mass of the k-th pole from the midst of the group of n poles (of n-th interaction).

The case n=4 corresponds with the hypothetical quark-lepton interaction with three generalized quark charges: b', g', r' and one lepton charge l .

So we have four charges b', g', r', l .

The case n=6 coorresponds with six charges of flavour and the case n=8 describes the eight gluon charges.

The complicated shape of strong interactions is implicated by the fact, that the strong interactions are in reality the superposition of three nonseparated interactions (n=3, n=6, n=8) with comparable forces.

The equation (14) both in general and in particular case when $Q = 0$ , will play meaning role in further considerations concerning the unification of interactions [25] .

2.4. In the case when $v > c$ (compare [1] ) the mass becomes complex mass and absolute mass becomes equal complex mass.

The equation (14) stands then in the form without module. All quantities in equation (7) are then expressed by complex number.

According to the equation (14), the gravitational mass, the electromagnetic mass, and the strong mass exist. All interactions give contribution to mass.

Though the electromagnetic and strong charges create usually the neutral object , they give the contribution to mass becouse of the module in equations (8) and (14).

Generally the equation describing total mass of the object has the shape:

$$M_{TOT} = \sum_{ijk} \alpha_{ijk} \left| Q_{ijk} \right| \tag{15}$$

i - describes all interactions

j - describes all kinds of charges of certain interaction

k - describes all poles of certain charge

When $v > c$ , there is no module in equation (15).

The weak interaction as the component of electroweak interaction possesses sources interacting with another charges (sources).

Generally, the principle of conservation of energy may be written in the shape:

$$\sum_i \alpha_i c^2 |Q_i| = E = \text{const} \tag{16}$$

The energy is a linear combination of modules of charges.

The kinetic energy is absorbed in the formula:

$$E_k = \frac{m_o c^2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and factor $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ is absorbed by coefficient $\alpha$ in equations (7) and (8).

In turn, the potential energy, in an attempt of mass treated as charge, corresponds with energy-mass of quantum of field in the procedure of quantisation of field.

Let's discuss the case of mass equal zero.
Let's analyse the formula:

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{17}$$

Let $m_o = 0$

If $v = c$, then on the right side of (17) we have undeterminated symbol $\frac{0}{0}$, which corresponds with any value of mass, in the case of photons, expressed by the formula: $m = \frac{h\nu}{c^2}$ ; $\nu \in R_+$ .

If $m_o = 0$ and $m = 0$, then $v < c$ or $v > c$ or even $v = 0$.
or $v$ expressed by complex number.

In the case of equations (7) and (14) $m_o=0$ means, that $\alpha \neq 0$, and $Q = 0$,
or $\alpha = 0$ and $Q$ any number.

The case of mass described by generalized quaternion will be discussed in one of the next works. [26]

## III

## Quantisation of gravitation

3.1. The equivalence of mass and charge described by formulas (7) and (8) must have consequences in General Relativity too.

Particurarly charge equivalent mass must give the contribution into energy-momentum tensor, which leads in consequences to modification of Einstein's equation.

$$R_{ik} - \frac{1}{2} g_{ik}R = T_{ik} \cdot \left(\frac{8\pi k}{c^4}\right) \tag{18}$$

$R_{ik}$ - the component of Ricci's tensor

$g_{ik}$ - the component of metric tensor

$T_{ik}$ - the component of energy-momentum tensor

$$R = g^{ik} R_{ik}$$

The contribution of charge to energy-momentum tensor is taken under consideration by introduction the generalized energy-momentum tensor, which is the sum of (hitherto existing) energy-momentum tensor $\mathcal{E}$ and mass-charge $Q$.

So we have:

$$T_G = Q + \mathcal{E} \tag{19}$$

or analogical equation for components of tensor

$$T_{Gik} = Q_{ik} + \mathcal{E}_{ik} \qquad (20)$$

and on the right side of equation (18) the component of tensor $T_{Gik}$ expressed by formula (20) exists now.

The equation (20) unifies interactions too and the mathematical structure of Einstein's equation is conserved. Naturally, the tensor Q must be symmetric.

We construct now the tensor Q in the following way:

$$Q = \begin{bmatrix} m & q_1^1 & q_1^2 & q_1^3 \\ & q_2^1 & q_2^2 & q_2^3 \\ & & q_3^2 & q_3^3 \\ & & & q_4^3 \end{bmatrix} \qquad (21)$$

We analyse matrix 4x4. We place the mass (the only pole of gravitational charge) into the highest position of first column.

The first pole of electric charge is placed into the highest position of second column and the second pole of electric charge is placed into the following lower position of the same column.

The following position of the third column (starting with the highest) are filled with the poles of charge of strong interaction : $q_1^2$, $q_2^2$, $q_3^2$ .

Analogically the fourth column is filled with poles of hypothetical, generalized quark-lepton interactions: $q_1^3$, $q_2^3$, $q_3^3$, $q_4^3$ .

We have:

$Q_{ik} = q_k^l \qquad l = 0,1,2,3; \quad k = 1,2,3,4; \quad i=l+1$

$q_k^l$ are generally the functions describing the spacetime contribution of $l,k$-th charge.

When $v < c$ , the functions $q_k^1$ must have been taken under consideration with module.

When $v > c$ , the functions $q_k^1$ may have complex values, what leads in consequences to appearing of complex components of tensor $g_{ik}$ and $R_{ik}$ existing in equation (18).

It means the doubling of the Einstein's equation to the number 20.
It corresponds with the results of M.J.Duff [27] where the complex tensor of curvature appears.

The curvature of space-time induced by mass of sources of other interactions (electromagnetic and strong and...) becomes integral part of the structure.

The equivalent of mass and charges results not only from equations (7) and (8) but exists in the structure of tensor Q, too.

The mixing of interactions is bound with it, that these same functions appearing in curvature tensor $R_{ik}$ and metric tensor $g_{ik}$ are connected both with mass by component of tensor $\mathcal{E}$ and with charge of some interaction by the component of tensor Q . The consequence of this fact is the increased knowledge about relations between gravitational, electromagnetic and strong interaction .

3.2. Let's write Dirac's equation with the shape:

$$(i\hbar\gamma^{\mu}\partial_{\mu} - \beta m_c)\psi = 0 \qquad (22)$$

$\psi$ - 4 spinor

$\gamma^{\mu}, \beta$ - Dirac's matrices 4x4

In our attempt $m_c$ is mass matrix 4x4 too. The mass matrix appeared earlier in the work [17, 28]
In our case it is matrix of effecive mass signalized already in the work [1] describing the effective interaction between object and arrangement.

The component of effective mass matrix is defined

$$m_{ij} = \hbar^2 \left( \frac{\partial^2 \varepsilon}{\partial k_i \, \partial k_j} \right)^{-1} \qquad i\ j = 1 \ldots 4 \qquad (23)$$

$m_{ij}$ may be complex number.
Naturally, the effective mass tensor is symmetric.

The generalized tensor of energy-momentum describes the interaction of objects and arrangement too, and the components of this tensor have the physical sense of mass.
So it may identify the effective mass tensor with generalized energy-momentum tensor.

So Dirac's equation has now the shape:

$$i\hbar\gamma^{\mu}\partial_{\mu}\Psi = \beta T_G \Psi \qquad (24)$$

and using the equation (18) we obtain

$$i\hbar\gamma^{\mu}\partial_{\mu}\Psi = \Theta\beta\,(R_{ik} - g_{ik}\ R)\,\Psi \qquad (25)$$

$$\Theta = \frac{c^4}{8\pi k}$$

Instead of generalized energy-momentum tensor, the Einstein's tensor (equal to it) has been put into Dirac's equation.

In this extremely easy way the quantisation of gravitation has been achieved. The essence of idea leading to solving of this fundamental problem is the interposition of one equation into the other!
The correctness of such procedure is supported by the fact, that the effects described both by General Relativity and by quantum mechanics, appear in neutron stars.

We take next the potential of external interactions under consideration and write equations (24) and (25) with the shape:

$$i\hbar\gamma^{\mu}\partial_{\mu}\Psi + V\Psi = \beta T_G \Psi \qquad (26)$$

$$i\hbar\gamma^{\mu}\partial_{\mu}\Psi + V\Psi = \Theta\beta\,(R_{ik} - g_{ik}R)\,\Psi \qquad (27)$$

V - 4-potential

In this way all interactions appear in Dirac's equation. They are introduced now into this equation not only by 4-potential, but by the effective mass tensor, which is equivalent to generalized energy-momentum tensor including the charges of all interactions.

The equations (26) and (27) connect classical and quantum descriptions and make possible connection of wave function with topology of space-time, and description of the quantum effects in curved space-time.

The existence of two types of potentials - an external 4-potential V and components of metric tensor $g_{ik}$ (each of these components have the sense of gravitational field) - makes possible to explain the quantum Titius-Bode law.

The Titius-Bode law informs, that the distance of the n-th planet from the Sun is expressed by the formula:

$$r_n = a + b\,2^n \qquad n \text{ - natural number} \tag{28}$$

This dependence is strictly quantum and shows the existence of quantum effects created by potential V in gravitational field.

The shape of Dirac-Einstein's equation (27) makes possible to explain this phenomenon, what is the succes of this attempt.

It is possible to connect in similar way Einstein's and Schrödinger's equations [29].

3.3. According to the considerations in chapter II, mass-charge tensor (or generalized energy-momentum tensor) must have been described by the matrix 8x8, because the 6-pole flavour charges or 8-pole gluon charges have been considered.
It corresponds with mass matrix 8x8 introduced by E.Cremmer [28]

Einstein's equation can be easily generalized for the case of 8 dimensions.
The dimension of metric tensor is equal to number of dimensions of the Universe.
Next the dimension of metric tensor must be equal to dimension of generalized energy-momentum tensor.

$g_{ik}$ may be described by 8-dimensional matrix.

In the equation:

$$R_{ik} = \frac{\partial \Gamma^{l}_{ik}}{\partial x^{l}} - \frac{\partial \Gamma^{l}_{il}}{\partial x^{k}} + \Gamma^{l}_{ik}\,\Gamma^{m}_{lm} - \Gamma^{m}_{il}\,\Gamma^{l}_{lm}$$

the identical procedure of summation may be done by the assumption that all the indicators have the values from set $A = \{1,2,3,\ldots 8\}$.

The identical situation occurs in the case of equation

$$\Gamma^{i}_{kl} = \frac{1}{2} g^{im}\left(\frac{\partial g_{ml}}{\partial x^{l}} + \frac{\partial g_{ml}}{\partial x^{k}} - \frac{\partial g_{kl}}{\partial x^{m}}\right)$$

The calculation

$$R = g^{ik} R_{ik}$$

may be done, when $i,k \in \{1,2,\ldots 8\}$ too.

Energy-momentum tensor

$$\mathcal{E}_{ik} = (p + \mathcal{E})\,\mu_i\,\mu_k - p\,g_{ik}$$

may be generalized for the case of 8 dimension.

Einstein's equation in 8-dimensional Universe corresponds with 8 dimensions described in the work [30], and with 8 dimensional mass matrix (8x8) occuring in the work by E.Cremmer [28], and with 8 dimensional matrix of energy existing in the work by E.Kane [31], too.

It is easy to construct Dirac's matrices 8x8.
Similarly 8-potential and 8-wave function may be introduced.

Four upper equations correspond with particle and antiparticle with spins: $\frac{1}{2}$ and $-\frac{1}{2}$.

Four down equations correspond with particle and antiparticle with spins $+\frac{1}{2}$ and $-\frac{1}{2}$ beeing supersymmetric equivalent of particle with spin 1.
(Supersymmetric equivalent has spin lower by $\frac{1}{2}$)

In this way the unification of interactions and quantisation of gravitation have been bound with the idea of supersymmetry at the cost of increasing of number of dimensions of Univers. In the light of the latest numerous works however it seems to be necessity.

The multiplication of Dirac's equations because of supersymmetry corresponds with 8x8 mass-charge tensor including 6 and 8-pole charges (and possibly others).

3.4. The in Dirac's equation occuring supersymmetric equivalents of particles with another spin than $\frac{1}{2}$ suggest the necessity of enlargement of formalism of Dirac's equation on the spins different than $\frac{1}{2}$.

It is worth to generalize Dirac's equation, because it contains the relativistic effects and may be used to description of particle with spin in any field of forces by connection with Einstein's equation.

It is easy to notice, that Dirac's equation describes both system and subsystem and then system is described in equation by external potential V, which in general case may be matrix too.

Such situation occurs in the case of nucleon where Dirac's equation may be written both for whole nucleon and for each of creating it quarks.

Dirac's equation may be easily generalized for the case of any value of spin $s=\frac{m}{2}$ $(m \in N \cup \{0\})$.

The situation of each $m$ is the same like in the case of $m = 1$ $(s=\frac{1}{2})$, the only difference is different complet of Dirac's matrices for each value of $m$.

In the general case the cellular shape of generalized Dirac's equation occurs.

$$\begin{bmatrix} \alpha_1 [D_{1/2}] & & & & & \\ & \alpha_2 [D_{2/2}] & & & & \\ & & \alpha_3 [D_{3/2}] & & & \\ & & & \alpha_4 [D_{4/2}] & & \\ & & & & \alpha_5 [D_{5/2}] & \\ & & & & & \ddots \end{bmatrix} \begin{bmatrix} \Psi_{1/2} \\ \Psi_{2/2} \\ \Psi_{3/2} \\ \Psi_{4/2} \\ \Psi_{5/2} \\ \vdots \end{bmatrix} = \begin{bmatrix} [E_{1/2}] \\ [E_{2/2}] \\ [E_{3/2}] \\ [E_{4/2}] \\ [E_{5/2}] \\ \vdots \end{bmatrix} \begin{bmatrix} \Psi_{1/2} \\ \Psi_{2/2} \\ \Psi_{3/2} \\ \Psi_{4/2} \\ \Psi_{5/2} \\ \vdots \end{bmatrix} \tag{29}$$

$D_{m/2}$ - Dirac's operator for spin $\frac{m}{2}$ in the shape of matrix 8x8.

$E_{m/2}$ - corresponding with $D_{m/2}$ Einstein's tensor equal generalize energy-momentum tensor.

$\Psi_{m/2}$ - 8-spinor

$\alpha_{m/2}$ - complex number (for any m) which multiplies each Dirac's operator creating different shapes of Dirac's matrices.

For each value of m , 8 Dirac's equations occur .
4 describe particles and antiparticles with spins $\frac{m}{2}$ and $-\frac{m}{2}$ , and 4 next describe supersymmetric equivalent of particle with spin $\frac{m+1}{2}$ . This supersymmetric particle has an antiparticle too, and both can exist in two spinal states $\frac{m}{2}$ and $-\frac{m}{2}$ .

Each particle existing in the spinal state $\frac{m}{2}$ may exist in the spinal state $\frac{n}{2}$ ; $n < m$ if $m > 0$ ; naturally $m-n = 2k$ ; k - natural number.
Such case is described by Dirac's equation for $\frac{n}{2}$ .

The necessity of introduction of supersymmetric particles is seen very clearly in the case of spin 0 .
There is the spinal degeneration: spinors $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ correspond with particle and antiparticle, but with what do correspond spinors $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ ?

And here the place for supersymmetric particles occurs. The supersymmetric particles with spin 0 are supersymmetric equivalent for particle with spin $\frac{1}{2}$ .

3.5.

The Dirac-Einstein equation can be generalized yet differently. It is necessary to answer the reproche that quantum electrodynamics describes quantum effects the best.
Really, the Rarita-Schwinger equation is the most general quantum equation

$$\epsilon^{\mu\nu\varrho\sigma} \gamma_5 \gamma_\nu D_\varrho \Psi_\sigma = 0 \tag{30}$$

The Rarita-Schwinger equation is the many member Dirac equation.
In each member there is the operator $D_\varrho = D_\varrho - m_\varrho$ (31)

$$m_\varrho = M + T + Q \tag{32}$$

M - mass matrix

T - energy-momentum tensor

Q - mass-charge tensor

(2) and (3) introduce the geometric effects by the Einstein equation, so the procedure of quantisation is the same as in the case of the Dirac-Einstein quantum equation.
Each member has different members described by the equation 3).
The generalization of the case of 8 dimensions is identical as in the case of the Dirac-Einstein equation.
This ranges the numbers 1...8 .
The gravitational Rarita-Schwinger equation may be obtained from the infinite number of squares (of a chequered pattern) - each of them corresponds with certain spin.

3.6. Dirac's matrices have to obey two conditions:

$$\gamma_i^2 = \underline{I}$$

$$\left[\gamma_i, \gamma_j\right] = 2\delta_{ij} \qquad (33)$$

The second condition may usually obtain easier shape:

$$\gamma_i\gamma_j = 0 \quad \text{for } i \neq j \qquad (34)$$

So we have two characteristic features of Dirac's matrices, norma and ortogonality, characteristic for wave function too.

We will next notice, that the original procedure of formulation of Dirac's equation treats Dirac's matrices as generalization of number [32], then we obtain that Dirac's matrices obeying conditions (30) and (31) may be interchanged by Dirac's matrices

$$\gamma = e^{i\varphi}\gamma$$

and we define module of the product of matrix and number:

$$\| aA \| = |a| A^2 \qquad (35)$$

a - complex number, $|a|$ - its module

In this way we are able to describe the fact that Dirac's matrix has the character of wave function in such a sense that the multiplication by complex number with module 1 doesn't change norma and ortogonality.

Really: when $\gamma_i^2 = \underline{I}$, then:

$$\left\| e^{i\varphi}\gamma_i \right\| = \left|e^{i\varphi}\right| \gamma_i^2 = \gamma_i^2 = \underline{I}$$

and when $\gamma_i\gamma_j = 0$, then $e^{i\varphi}\gamma_i \cdot e^{i\varphi_j}\gamma_i = e^{i\varphi_i}\, e^{i\varphi_j}\gamma_i\gamma_j = 0$

All Dirac's matrices in certainccell of equation (29) must have been multiplicated by the same factor $\alpha_j = e^{i\varphi}\gamma_j$ .
The purpose of this work is not calculation of each $\alpha_j$ .

3.7. Quantisation of gravitation described with Dirac-Einstein equation explains at once problem of spin and negative self-energy in quantum gravity. It may be used to description of particle or system with each spin or system composed of particle with different (any) spins.
We pay great complication by the solving of Dirac-Einstein equation for great beauty and simplicity of idea.

## IV
## Recapitulation

4.1. The equations (7) and (8), which are the main idea of unification of interactions don't express the idea too easy. The simplicity is an argument supporting after all.

The equations with the shape $m = \alpha q$ occure moreover in other works.

M.J.Duff [24] considers low-energetic four-dimensional heterotic strings corresponding with p-branes (p=0,1,2) characterized by mass of unit of p-volume $M_{p+1}$ and topological "magnetic" charge $g_{p+1}$ .

Authors consider situation when

$$\sqrt{2}\, k\, M_{p+1} = g_{p+1}$$

G.W.Gibbons and M.J.Perry [33] analyse an infinite ladder of elementary states with mass $m_n$ and charge $e_n$ bound by dependence:

$$\sqrt{2}\, k\, m_n = e_n$$

and analogical sequence of soliton state with mass $\tilde{m}_n$ and magnetic charge $g_n$ conncected by analogical equation

$$\sqrt{2}\, k\, \tilde{m}_n = g_n$$

Moreover in the work by M.J.Duff and J.X.Lu [23] the same equation binding electric charge with mass

$$e = \sqrt{2}\, k\, m$$

occurs at the attempt of description of compactification concerning the conversion of particle in D=10 to D=4 dimensions.

M.J.Duff [34] uses equations (7) and (8) in the form

$m^2 \sim Q^2$ , too.

Nevertheless all mentioned authors didn't notice the general character of analysed equation.
It had not been interpreted in the full sense with all consequences for unification of interactions.
The trial of enlargement to other kinds of charges had not been undertaken.

It is worth to add, that equations (7) and (8) had been obtained in this work totally differently than in cited works and here the start point was Dirac's conception of unempty vacuum.

The presented conception of quantisation of gravitation does not impairs the efforts of other authors.
I agree with idea reflected in the work by M.J.Duff [27] that Reality may be described by plenty of theories perhaps.

We can not exclude, that before the discovery of one, most general theory, we ought to know plenty of theories.

The way to Unity is Variety.
Variety is manifestation of Unity.

4.2. This work is an introduction to further investigation. First of all it is necessary to bind conceptions from this work with results of: A.Ashtekar, Y.Bars, E.Cremmer, M.J.Duff, J.Goldstone, E.Witten and others.

The exploration leading in this direction will be the subject of next work [35] .

Moreover it is worth to generalize Schrödinger equation in the case of more dimensions, which will be described in the work [29] and to bind the conceptions presented in this text with conceptions showed in previous works: [1] and [36] .

It is necessary to investigate the structure of unempty vacuum and the effects bound with it, what will be done in the work [26].

The refering to periodic structure of space-time appearing in the work by M.J.Duff will be especially interesting.

It is necessary to consider numerous cases, when number of dimensions of Univers is bigger than 4, for example 10,11,26 and to investigate what consequences an attempt from the present work implicates, where number of analysed dimensions is equal 8 .

Author is convinced that real space has more dimensions than 4.

It is worth to consider the situation, when the components of energy-momentum tensor are described with quaternions or generalized quaternions.

An attempt presented in this work is one from many attempts to problems of unification of interactions and quantisation of gravitation.

Its advantage is simplicity and compactness. Presented results create new challenge of binding of these ideas with other conceptions of unification of interactions and quantisation of gravitation.

In this work the problem of unification of interactions has been solved, which Einstein had not time to do.

What is more, the pattern of unification of all interactions has been created, so as the Mendeleev's table contained the not yet discovered elements.

Parallelly, in purpose to realize this task, the quantisation of gravitation had to be made, and so the controverse Einstein against Bohr has been solved. Einstein was right, because quantum mechanics is not the final theory, but Bohr was right, too, because this theory is valuable (it contains the "healthy" nucleus - the Dirac equation).

This work, together with few other works by this author, (for example [1] or [36]), creates a new vision of physics but based strongly on the foundation created by de Broglie, Dirac, Einstein, Maxwell and implicated in the logical way by earlier accepted ideas.

References

[1] Z.Morawski, 'The Implications of Complex Mass', in publishing
[2] E.Witten, Nuclear Physics B186 (1981) p412
[3] E.Witten, Nuclear Physics B195 (1982) p481
[4] E.Witten, Physics Letters vol.155B no3(1985) p151
[5] M.J.Duff, T.Inami, C.N.Pope, E.Sezgin, K.S.Stelle, Nuclear Physics B297 (1988) p515
[6] M.J.Duff, J.X.Lu, Nuclear Physics B347 (1990) p394
[7] M.J.Duff, K.S.Stelle, Physics Letters B vol.253 no1,2(1991) p113
[8] P.Goddard, J.Goldstone, C.Rebbi, C.B.Thorn, Nuclear Physics B56 (1973) p109
[9] D.Olive, J.Scherk, Physics Letters vol.44B no3(1973) p296
[10] L.Brink, D.Olive, Nuclear Physics B56 (1973) p253
[11] E.Cremmer, B.Julia, Nuclear Physics B159 (1979) p141
[12] E.Cremmer, B.Julia, J.Scherk, Physics Letters vol.76 no4 (1978) p409
[13] A.Ashtekar, C.Ravelli, L.Smolin, Physical Review D vol.44 no6 (1991) p1740
[14] A.Ashtekar, G.T.Horowitz, Physical Review D vol.26 no12 (1982) p3342
[15] Y.Bars, Nuclear Physics B317 (1989) p395
[16] J.Goldstone, Il Nuovo Cimento vol.XIX N.1 (1961) p154
[17] J.Goldstone, A.Salam, S.Weinberg, Physical Review vol.127 no3 (1962) p965
[18] B.Badełek, J.Kwieciński, Reviews of Modern Physics vol.68 no2 April 1996 p445
[19] B.Badełek, K.Charuła, M.Krawczyk, J.Kwieciński, Reviews of Modern Physics Vol.64 no4 October 1992 p927
[20] D.L.Smith, C.Maihiot, Reviews of Modern Physics Vol.62 no1 January 1990 p173
[21] J.D.Björken, S.D.Drell, Relativistic Quantum Mechanics; McGraw-Hill Inc.
[22] E.Leader, E.Predazzi, An Introduction to Gauge Theories and the 'New Physics'; Cambridge University Press (Chapter IV)
[23] M.J.Duff, J.X.Lu, Nuclear physics B416 (1994) p301

[24] M.J.Duff, Ramzi R.Khuri, Ruber Minasion, Nuclear physics B418 (1994) p195

[25] Z.Morawski, 'The Particles and Lattices', in preparation

[26] Z.Morawski, 'The Considerations about Mass and Spacetime', in preparation

[27] M.P.Blencowe, M.J.Duff, Nuclear Physics B310 (1988) p387

[28] R.Cremmer, J.Scherk, J.H.Schwarz, Physics Letters vol.84B no1 (1979) p83

[29] Z.Morawski 'The Equation of Phases', in preparation

[30] M.J.Duff, J.X.Lu, Nuclear Physics B354 (1991) p141

[31] E.O.Kane, Pergamon Press 1957 Vol.1 p249

[32] P.A.M.Dirac, Proc.Roy.Soc. A109 (1926) p561

[33] G.W.Gibbons, M.J.Perry, Nuclear Physics B248 (1984) p629

[34] M.J.Duff, Nuclear Physics B459 (1996) p125

[35] Z.Morawski, 'Strings, Loops and Membranes', in preparation

[36] Z.Morawski, 'Mechanism of High Temperature Supercon- duction' in publishing